N-Coloring of 2-Edge Colored Planar Graphs

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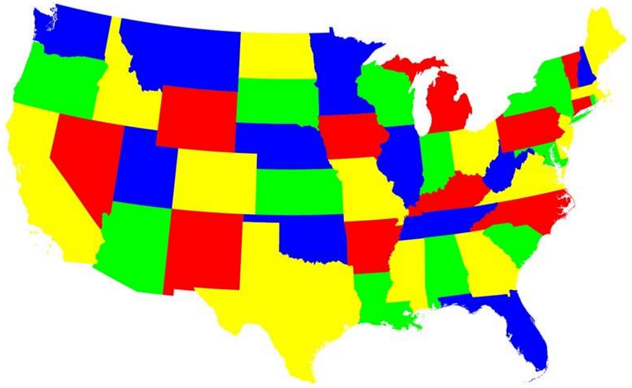
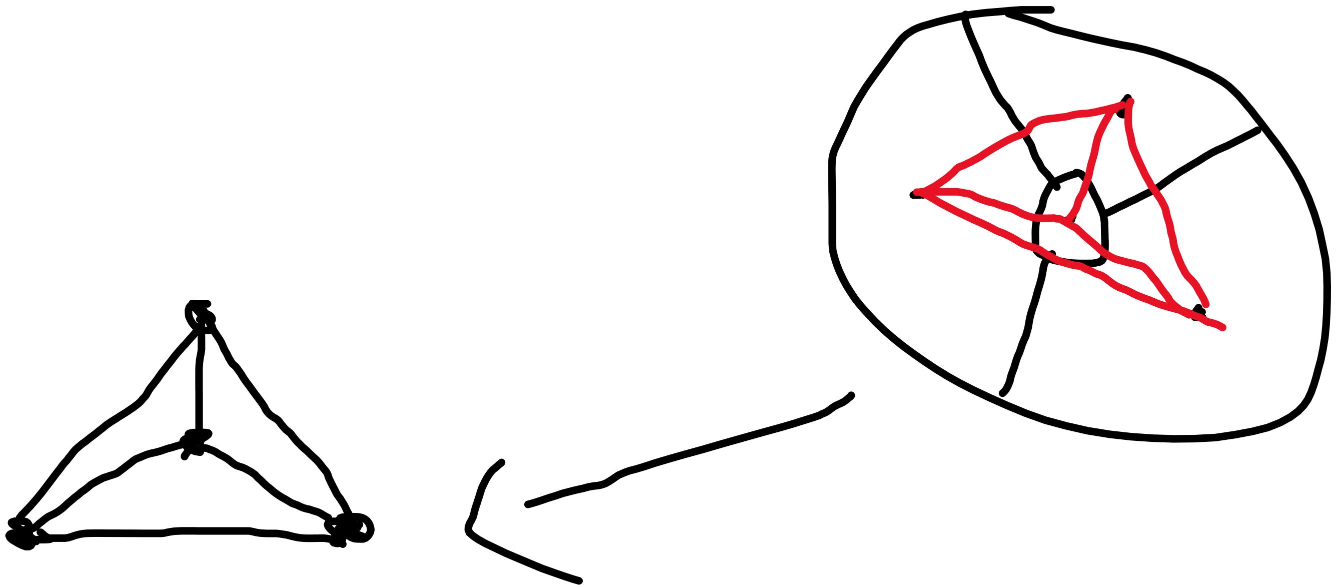
A planar graph is a graph that can be embedded into the plane such that no 2 edges intersect.



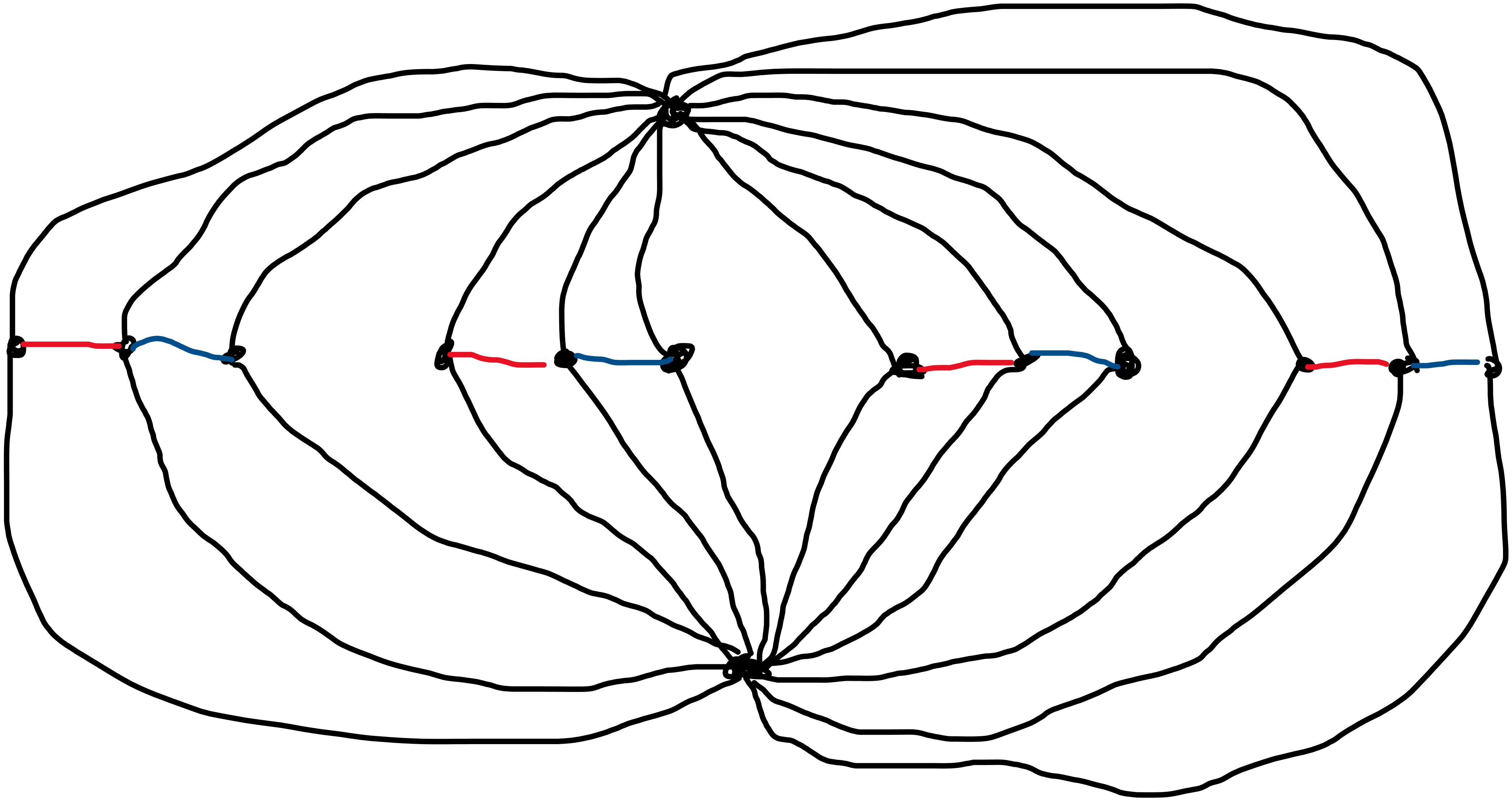
Four coloring theorem: Every planar has a proper coloring in four colors



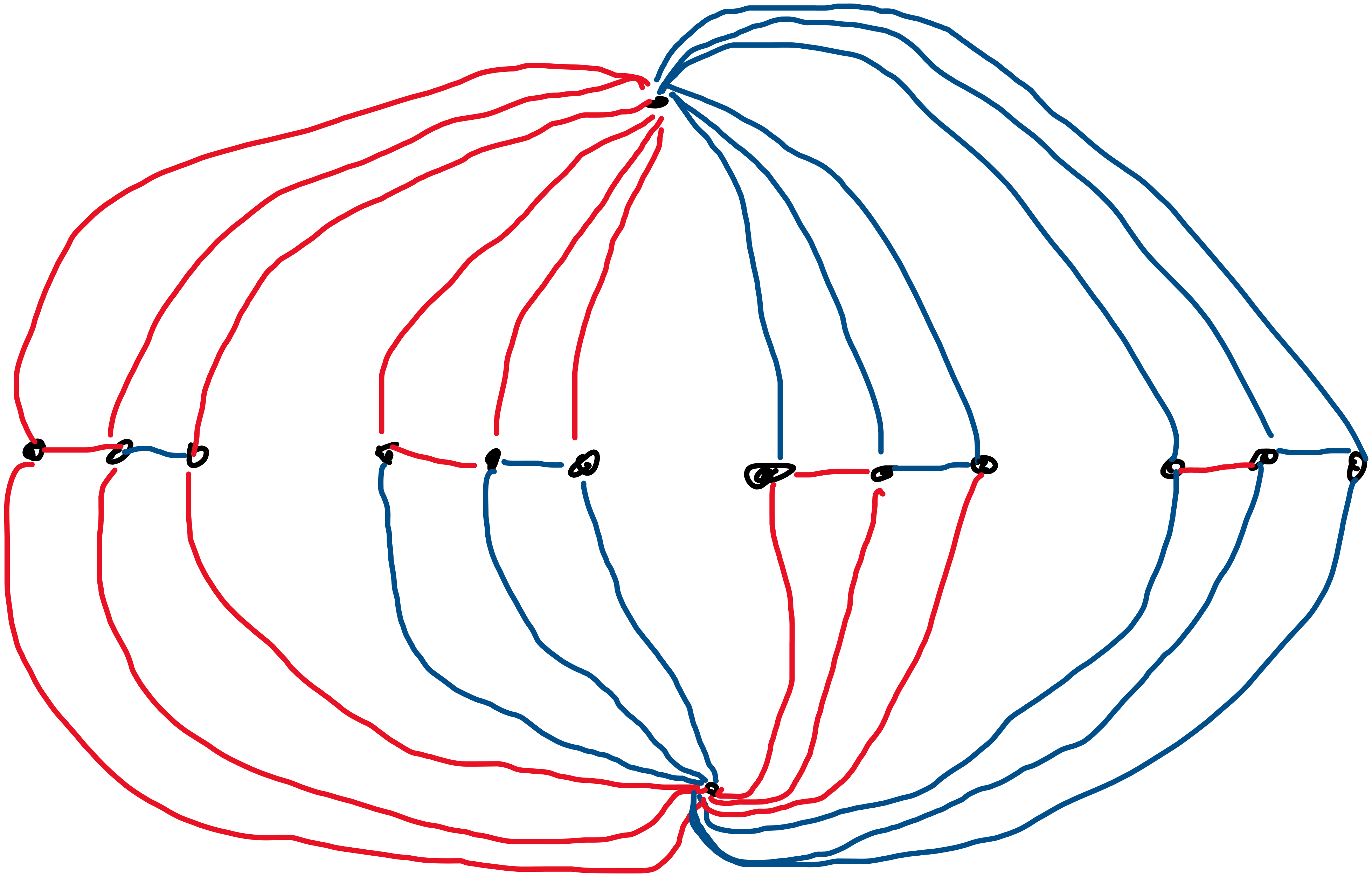
Constructing a 2-Edge colored graph colorable in no less than 14 colors

First, we will start with four groups of bichromatic 2-paths.

We then add 2 more vectices to this graph which will each be connected to the existing vertices of the graph

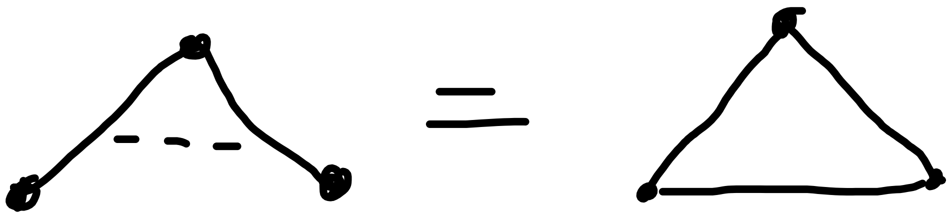


Now we will assign this coloring to these edges



With a stricter definition of a planar 2-edge colored graph we can easily find an upper bound for the chromatic number.

Any bichromatic 2-path is equivalent to adding an edge between the vertices on opposite ends. That is to say:



If a graph is planar after adding in these edges for all bichromatic 2-paths, we will says that this graph has only planar bichromatic 2-path

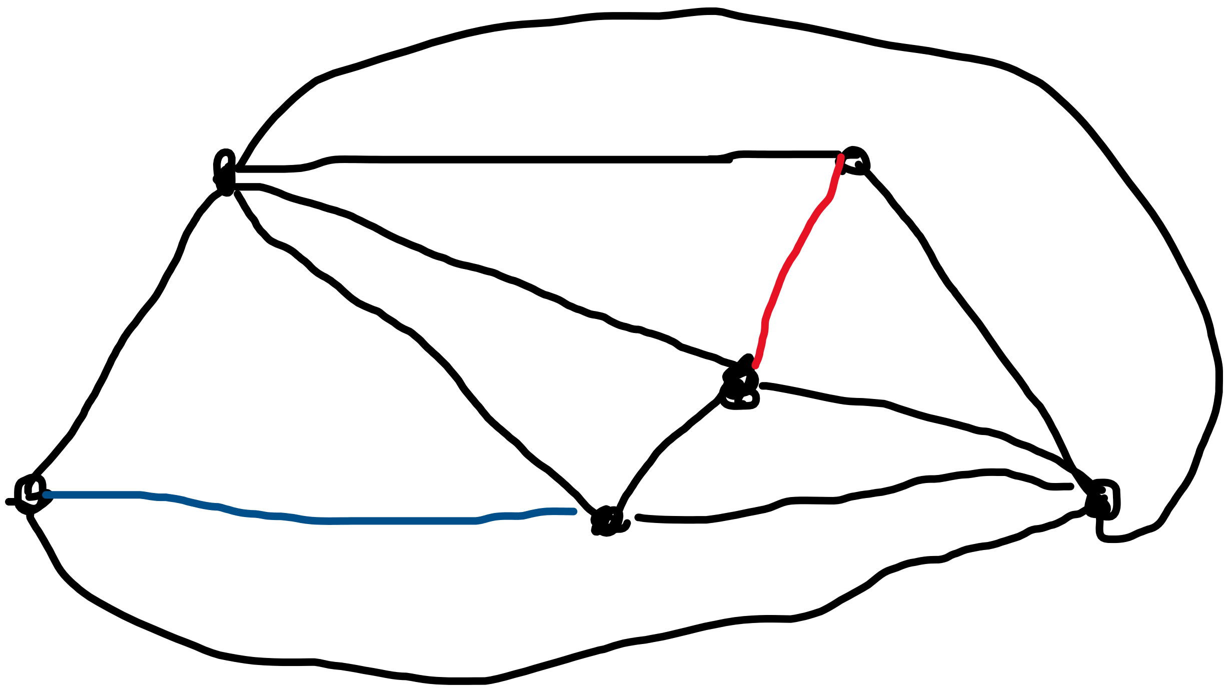
Theorem: A planar mixed 2-edged colored graph G containing only planar bichromatic 2-paths is colorable in 7 colors.

Proof

We can first remove the edges between edges that share a vertices replacing them with the corresponding edge between the vertices at opposite ends of the 2-path to obtain a modified graph G'. Now we will consider the graph obtained by replacing the edges of G' with vertices and adding an edge between the vertices corresponding the edges with edges between them. Since there are only edges between edges of opposite colors, this is a bipartite graph and so is colorable in 2 colors. Let f(e) be a function mapping an edge e to it's color under some fixed coloring such that the colors are in the set {-1,1}.

Since G' is planar, there is a coloring g of the uncolored graph in 4 colors 0,1,2,3. Now we will construct a new coloring such that every vertex v will be colored g(v) if it has no edges colored -1 under f connected to it and -g(v) otherwise. Since no edges sharing a vertex have an edge between them, this is a valid coloring of the unsigned graph of G' and for any 2 edges with an edge between them, they receive colors of the opposite sign and so the edges are not colored the same.

Mixed 2-edge colored graph containing only planar bichromatic 2-paths colorable in no less than 5 colors



References

Montejano, A. et. al. Homomorphisms of 2-edge-colored graphs, Discrete Applied Mathematics, https://doi.org/10.1016/j.dam.2009.09.017